18

Testing of Hypothesis

18.1 INTRODUCTION

The estimation of population parameters and the testing of hypothesis concerning those parameters are similar techniques, but at the same time there are major difference in the interpretation of results arising from each method. When we are concerned with measurement, say, of expenditure on entertainment, the appropriate methods will be the process of elimination. Hypotheses using is very helpful in examining the validity or otherwise of theories such as wage increase leads to rising prices, advertisements leads to increase amount of sales.

18.2 CONCEPT OF HYPOTHESIS : Any statement or question about a statistical population on the values of its parameters is called a statistical hypothesis. The procedure which enables us to examine whether a certain hypotheses is true or not is called testing of hypothesis. For this purpose, we have to collect relevant information, and process it using statistical techniques and then test the above statement. In general formulation of hypothesis is generally concerned with the causes of a certain phenomenon or a relationship between two or more variables under estimation.

18.2.1 Concept of Null hypothesis: A null hypothesis can be defined as a statistical hypothesis which states that there are no differences between the observed and expected data. It is a statement about a population parameter and the test is used to decide whether or not to accept the hypothesis. A null hypothesis is denoted by H_0 is always one of status quo or no difference. When the statement of the null hypothesis is false, some thing else must be true. In view of this probability, when ever a null hypothesis is specified an alternative hypothesis denoted by H_1 should be mentioned. H_1 is the opposite statement of H_0. So when one hypothesis is accepted, it implies rejection of the other.

18.2.2 Level of significance and critical region:

The decision about rejection or acceptance of the null hypotheses is based on probability concept. Assuming the null hypothesis to be true we calculate the probabilities of obtaining a different equal to or greater than the observed difference. If this probability is found to be small say less than 0.05, then conclusion is that the observed value of the statistic is rather unusual and has been arisen because the underlying assumption (i.e. null hypothesis) is not true. In this case, we conclude that the observed difference is significant at 5% level of significance. However, when probability is not very small, say more than 0.05, the observed difference cannot be considered to be unusual and is attributed to sampling fluctuation only.

The set of values of the test statistic which leads to rejection of the null hypothesis is called critical region of the test and the probability with which a true null hypothesis is rejected by the test is known as 'size' of the critical region.

18.3 TAILS OF A TEST:

In statistics, the rejection region in hypothesis testing can be on both sides of the probabilities more with the non-rejection region in between the two rejection regions. A hypothesis test with two rejection region is called a two tail test and a test with one rejection region is called a one-tail test. The one rejection region can be on either side-right or left. If the rejection region is on the right side of the above, then the test is known as the right-trail test. When the test has a rejection region on the left side, then it is called a left tail test.

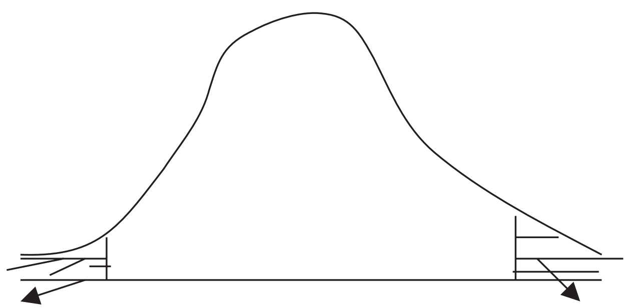
For example θ be any population parameter whose hypothesised value be θ_0, we have constructed

$H_0 : \theta = \theta_0$ against

$H_1 : \theta = \theta \neq \theta_0$

which is shown below:

he shaded area on either side is $\alpha / 2$



Rejection region   = $_0$Rejection region

Non

Rejection

Area

(Two tail test)

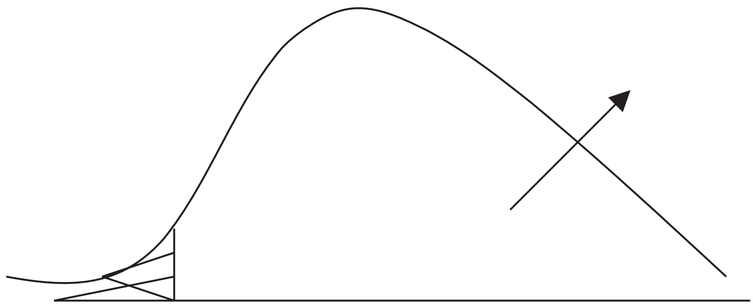
On Tail test

We set $H_0 = \theta = \theta_0$

$H_1 = \theta < \theta_0$

Then we show the following diagram

Non rejection region

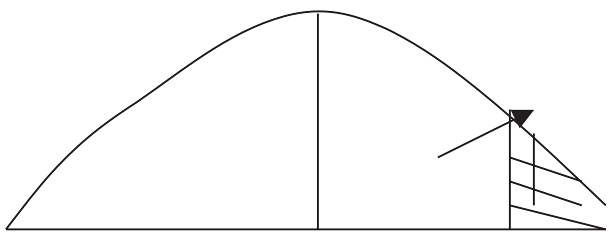


Rejection region

(example of a right-tail test)

18.4 PROCEDURE IN HYPOTHESIS TESTING:

There are five steps involved in testing on hypothesis which are as follows:



(a) Formulate a Hypothesis: The first step is to set up two hypothesis instead of one in such a way that if one hypothesis is true, the other is false. Alternatively; if one hypothesis is false or rejected then the other is true or accepted.

(b) Set up a suitable significance level: After formulating the hypothesis, the next steps is to test its validity at a certain level of significance. The confidence with which a null hypothesis is rejected or accepted depends on the significance level used for the purpose.

(c) Select test criterion: The next steps in hypothesis testing in the selection of an appropriate statistical technique as a test criterion. There are many techniques from which one is to be chosen. For example, when the hypothesis partners to a large of more than 30, the Z test implying normal distribution is used for population mean. If the sample is small ($n<30$) the t test will be more appropriate. The test criteria that are frequently used in hypothesis testing are Z, t, f and x^2.

(d) Compute: After selecting the sampling technique to less the hypotheses, the next step includes various computations necessary for the application of that particular test. These computations include the testing statistic as also its standard error.

(e) Making decision: The final step in hypothesis testing is to draw a statistical decreases, involving the acceptance or rejection of the null hypothesis.

18.5 HYPOTHESIS TESTING FOR POPULATION MEAN:

(a) Large samples

Critical values of Z for selected levels of significance

Level of significance ($\propto$)	0.10	0.05	0.01	0.005
Two Tail Critical values of |Z|	1.645	1.96	2.58	2.81
Right tail Critical value of Z	1.28	1.645	2.33	2.58
Left tail Critical value of Z	-1.28	-1.645	-2.33	-2.58

e.g. (1) In order to test whether average weekly maintenance cost of a fleet of buses is more than Rs. 500, a random sample of 50 buses was taken. The mean and the standard deviation were found to be Rs. 508 and Rs. 40 (Use $\alpha = 0.05$)

Solution

We set up $H_0 : \mu = 500$

Against $H_0 : \mu > 500$

The significance level is 0.05 and H_1 is a right tail one

The test criterion is the Z test.

Computations: $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{40}{\sqrt{50}} = 5.657$

$$z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{508-500}{5.657} = 1.414$$

At $\propto = 0.05$, the critical value of Z = 1.645 (i.e. right tail) and the computed value of Z = 1.414 does met fall in the rejection region, we accept null hypothesis H_0 that ì = 500. One conclusion is that the average maintenance cost is not more than Rs. 500.

(2) The quality control department of a processing company specifies that the mean net weight per pack of its produce must be 20 gms. Experience has shown that the weight are approximately normally distributed with a S-d, of 1.5 gms. A random sample of 15 packs yield a mean weight if 19.5 gms. Is this sufficient evidence to indicate that the true mean weight of the packs has decreased (use 5% significance levels).

Solution

We set up H_0 = ì = 20 against

H_1 = ì < 20

The significance level is 0.05 and H1 is a left tail one.

The test criterion is the Z test as n< 30 but s is known

Computations: $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{40}{\sqrt{50}} = 5.657$

$$\therefore \quad z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{19.5 - 20}{0.387} = -1.29$$

At 0.05 level of significance the critical value of Z (Left tail) is -1.645.

As it can be seen that computed Z does not fall in the rejection region.

So we accept H_0 and reject H_1. own conclusion is these no sufficient evidence to indicate that the true mean has decreased.

(3) A company is engaged in the packaging of a suppressing quantity tea in jar of 500 gm each. The company is of the view that as long as jars contains 500 gms of tea, the process is in control. The standard deviation is 50 gm. A simple of 225 jars is taken at random and the sample average is found to be 510 gm. Has the process gone out of control.

Solution

We set up $H_0 : \mu = 500$ gms

Again $H_0 : \mu \neq 500$ gms

In order to solve the problems, we have to assume a level of significance, $\alpha = 0.05$.

The computation for the test is as follows

$$z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{510 - 500}{\frac{50}{\sqrt{225}}} = 3$$

For two sided Critical region, Z $\alpha/2$ = 1.96 for $\alpha = 0.05$ (Table). Here computed value of Z does not fall in the critical region so we accept H_0 and reject H_1. This means process is not in control.

(b) Hypothesis test about a population mean for small samples:

The Z test used earlier, is based on the assumption that the sampling distribution of the mean is a normal distribution. This is applicable when the sample is large that is its size is at least 30, when a sample is small, the assumption of normal distribution does not whole goods an as such, the Z test will not be appropriate. Instead another test known as t-test used. The procedure of testing the hypothesis is the same except that instead of Z value, the t value is used. Also for tie as sample be less than 30, population SD σ be known we have to null Z test. In t test $\sigma_{\bar{x}}$ = S÷ $\sqrt{n-1}$ Where S is the sample Sd. Actually when s is unknown, we estimate it using unbiased estimator as $\hat{\sigma} = \sqrt{\dfrac{\sum(x-\bar{x})^2}{n-1}}$, but S $= \sqrt{\dfrac{\sum(x-\bar{x})^2}{n}}$ and $\sigma_x = \dfrac{\sigma}{\sqrt{n}}$, So we get $\sigma_x = \dfrac{s}{\sqrt{n-1}}$

e.g. (i) Ten individuals are chosen at random from a population and their height are found to be (inches) 63, 63, 66, 67, 68, 69, 70, 70, 71 and 71. In the light of this data, there is a suggestion that the mean height in the population is 66. Use 5% significance level for testing purposes.

Solution:

We set H_0 : μ= 66" against

H_1: μ≠66"

x	$x-\bar{x}$	$(x: \bar{x})^2$
63	-4.8	23.04
63	-4.8	23.04
66	-1.8	3.24
67	-0.8	0.64
68	0.2	0.04
69	1.2	1.44
70	2.2	4.84
70	2.2	4.84
71	3.2	10.24
71	3.2	10.24

$$\bar{x} = \frac{678}{10} = 67.8$$

$$\sigma = \sqrt{\frac{\sum(xi-\bar{x})^2}{n-1}} = \sqrt{\frac{81.60}{9}} = \sqrt{9.066} = 3.011$$

$$\therefore \sigma_x = \frac{\sigma}{\sqrt{n}} = \frac{3.011}{\sqrt{10}} == 0.95$$

We compute $t = \dfrac{\bar{x}-u}{\sigma/\sqrt{n}} = \dfrac{67.8-66}{0.95} = 1.89$

Two tailed table value of t with 9d. σ f at 5% level is found to be 2.26.

As computed value as t < tabulated 't'. So computed t does not lie in the critical region. We accept H_0 and conclude that the experiment provides no ground for doubt that the mean height is 66".

e.g.(2) A soap manufacturing company was distributing a particular brand of soap through a large number of retail shops. Before a heavy advertisement campaign, the mean sales per week per shop was 140 dozens. After the campaign, a sample of 26 shops was taken and the mean sales was found to be 147 dozens with standard deviation 16. What conclusion do you draw on the impact of advertisement on sales? Use 5% significance level.

Solution

We set up $H_0 = \mu = 66" = 140$

Again $H_1 = \mu \neq 140"$

It is given S = 16, n = 26, $\bar{x}$ = 147

$$\sigma_{\bar{x}} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{s}{\sqrt{n-1}} = \frac{16}{\sqrt{25}} = 3.2$$

Now $t = \dfrac{\bar{x} - u}{\sigma_{\bar{x}}} = \dfrac{147 - 140}{3.2} = 2.19$

From the table, for (26-1) = 25 d.o.f., $t_{005} = 2.06$. Since Computed value of t> t_{005}, we reject the null hypothesis. That is, the advertisement many be considered to have changed the average sales volume.

18.6 HYPOTHESIS TEST CONCERNING PROPORTION:

A random sample of size n shows that the proportion of numbers processing certain attributes (e.g. defectiveness or un biased ness etc.) is P. it is required to that the hypotheses that the proportion P in the population is equal to specific value P_0 i.e. $H_0 = (P = P_0)$.

The various procedures given for hypothesis testing of mean still hold good except we assume the normality of the population and the normal approximation of sampling distribution of the sample proportion p. In this test we use test static $Z = \dfrac{P - P_O}{\sigma_P}$

Where $\sigma_P = \sqrt{\dfrac{P_O(1-P_O)}{n}}$ or $\sqrt{\dfrac{P_O Q_O}{n}}$

e.g. The product manager wishes to determine whether or not to change the package design for her product. She feels that it will be worth considering only if more than 60% of the non-users prefer the new box to the old one. She selects or random sample of 100 persons who are non-users and finds that 73 persons prefer the new box. Should she change the design? (Significance level = 0.05)

Solution

We set up $H_0 = P = 0.6$

Against $H_1 = P > 0.6$

Significance level $\alpha = 0.05$ implies $Z_{0.05} = 1.645$ as the test is one-tailed.

Sample proportion P = $\frac{73}{100}$ = 0.73

$$\sigma_P = \sqrt{\frac{0.6x0.4}{100}} = 0.04898$$

Computed Z = $\frac{0.73 - 0.6}{0.04898} = 2.65$

As computed Z > Tabulated value as Z (= 1.645)

Since the decision rule suggest that the null hypothesis should be rejected, we can conclude that the sample gives sufficient evidence that more than 60% of the non-users prefer the new box design and the product manger should change the package design of her product at the specified significance level.

Exercise:

(i) A manufacturer for a new motorcycle claims for it an average mileage of 60 km/the with a standard deviation of 2 km under city conditions. However, the average mileage is 64 trials is found to be 57 km. is the manufacturer's claim justified.

Answer: Z = -12. Manager's claim can met be justified]

Test of Significance (1)

1. A sample of size 400 was drawn and the sample mean was found to be 99. Test whether the sample could have come from a normal population with mean 100 and SD 8 at 5% level of significance.
2. A sample of 1,000 students from Bombay university was taken and their average weight as found to be 112 pounds with a standard deviation of 20 ponds. Could the mean weight of the students in the university be considered to be 120 pounds?
3. A sample of 400 items is taken from a normal population whose mean is 4 and whose variance is also 4. if the sample mean is 4.45, can the sample be regarded as a truly random sample?
4. The mean IQ of a sample of 1,600 children was 99. is it likely that this was a random sample from a normal population with mean IQ 100 and SD 15?

5. A random sample of 200 tins of coconut oil gave average weight of 4.95 kgs. With a SD of 0.21. Do we accept the hypothesis of net weight of 5 kgs. Per tin at 1% level of significance?

6. An automatic machine was designed to pack exactly 2 kg. of vanaspati. A sample of 100 tins was examined to test the machine. The average weight was found to be 1.94 kg. with SD 0.10 kg. is the machine working properly?

7. A Company has been producing steel tubes of mean inner diameter of 2.00 cms. A sample of 10 tubes gives a mean inner diameter of 2.01 cms and variance of 0.004 sq. cms. Does the sample mean significantly differ from the actual mean?

8. Ten packets are taken at random from an automatic filling machine. The mean net weight of the 10 packets is 11.8 kg. and SD is 0.15 kg. Does the sample mean significantly differ from the intended weight of 12 kg.?

9. A random sample of size 20 from a normal population gives a sample mean of 42 and sample SD of 6. Test the hypothesis that the population mean is 44. State clearly the alternative hypothesis you allow for and the level of significance adopted.

10. Ten individuals are chosen at random from population and their heights are found to be (in inches); 63, 63, 66, 67, 68, 69, 70, 70, 71, and 71. in the light of data, discuss the suggestion the mean height of the population is 66 inches.

11. A fertilizer mixing machine is et to give 12 kg. of nitrate for every quintal bag of fertilizer. Ten 100 kg. bags are examined. The percentages of nitrate are 11, 14, 13, 12, 13, 14, 11, and 12 respectively. Is there any reason to believe that the machine is defective?

12. A random sample of 16 values from a large population showed a mean of 41.5 and sum of squares of deviations from the mean equal to 135. Can it be assumed that the mean of the population is 43.5?

13. A random sample of size 10 was taken from a normal population whose variance is known to be 7.056 sq. inches. If the observations are 65, 71, 64, 71, 70, 69, 64, 63, 67 and 68. Test the hypothesis that the population mean in 69 inches. Use 5% level of significance.

14. A dice was rolled 400 times and 'six' resulted 80 times. Do the data justify the hypothesis of an unbiased dice?

15. In a sample of 500 from a village in Bihar. 280 are found to be wheat eaters and the rest rice eaters. Can we conclude that the food articles are equally popular.

16. In a random sample of 400 persons from a large population 120 are female. Can it be said that males and females are in the ration 5:3 in the population? [Use 10% significance level.]

17. In a survey of 70 business firms. It was found that 45 were planning the expand their capacities next year. Does it sample information contradict the hypothesis that 70% of the firms in general are planning to expand next year?

18. Certain experimental breeding shows 5321 yellow peas and 1804 green peas. The expectation is 25% green peas on a scientific hypothesis. Can this divergences from the expected value have arisen from sample fluctuations only?

19. In a sample of 500 people in Kerala 280 are tea drinkers and the rest are coffee drinkers. Can you assume that both coffee and tea are equally popular in this state. Use 1% level.

Test of Significance (2)

1. A new variety of potato grown on 250 plots gives rise to a mean yield of 82 tons per acre with a SD of 14.6 tons per acre. Is it reasonable to assert that the new variety is superior in yield to the standard variety with a established yield of 80.2 tons per acre.

2. A machine part was designed to withstand a minimum average pressure of 120 units. A random sample of size 100 from a large batch was tested and it was found that the average pressure which these parts can withstand is 105 units with a SD of 20 units. Test whether the batch meet the specification.

3. A manufacturer of ball point pens claim that a certain pen, he manufactures has a mean writing life of 400 pages with a SD of 20 pages. A purchasing agent selects a sample of 100 pens and them to test. The mean writing life for the same was 390 pages. Should the purchasing agent rejects the manufacture's claim?

4. The average number of defective articles in a certain factory is claimed to be less than the average for all the factories. The average for all the factories is 30.5. A random sample of 100 defective articles showed sample mean to be 28.8 and SD to be 6.35. Test the claim of the factory.

5. The mean life of a sample of 400 fluorescent light bulbs produced by a company is found to be 570 hours with a SD of 150 hours. Test on the basis of this data whether mean life of the bulbs manufactured by the company is 1,600 hours. (Use 1% significant level).

6. it is claimed that students entering a college have an average IQ higher than 100. A random sample of 16 is taken and the sample mean is found to be 106. The sample SD is 10. is the claim supportable? [for 15 degrees of freedom p $(t>1.75) = 0.05$]

7. The highest of ten children selected at random from a given locality had a mean 63.2 cm and variance 6.25 cm. Test at 5% level of significance that children of the locality are on average less than 65 cm in all.

8. The heights of 10 students of college are 70, 70, 62, 67, 61, 68, 64, 66, 64, 68 inches. Is it reasonable to believe that average height of all students of the college is greater than 64 inches?

9. The mean weekly sales of the chocolate bar in a candy store was 146.3 bars per store. After an advertisement campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and a SD of 17.2. Was the advertisement successful?

10. A sample of six bags gave the following weights in ('00) gms. 37, 25, 34, 52, 30, 20. Test whether the weights of the bags is below the average expected value of 3,500 gms or not.

11. The weight of 10 machine parts from a large consignment were fond to be 670, 700, 620, 680, 610, 680, 700, 640, 640, 660 gm. Is it reasonable to believe that the average weight of the machine part is greater than 640 gm? [Given t = 1.83 for 9d of f at 5% level of significance]

12. In a sample of 400 burners, there are 12 burners whose internal diameters are not within the tolerance limit. Is this a sufficient evidence of contradicting that the manufacturing process is turning out to be more than 2% defective?

13. In a big city 325 men out of 600 men were found to be smoker. Does this information support conclusion that the majority of men in the city were smoker?

14. The manufacturer of a patent medicine claimed that it was 90% effective in relieving an allergy for a period of 8 hrs. in a sample of 200 people who had the allergy, the medicine provide relief for 160 people. Determine whether the manufacture's claim is legatine.

15. In a sample 400 parts manufactured by a factory, the number of defective parts was found to be 30. The company, however, claimed that only 5% of their product is defective. Is the claim tenable?

16. A manufacturer claimed that at least 95% of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test his claim at 5% level of significance.

17. A candidate in a election form a large constituency thinks that he will win the election if at least 45% of the electorate vote for him. He therefore conducts sample survey to enable him to decide whether he should stand for the election or not. The survey covers 10,000 voters and it is found that 4,420 voters would vote for him. Advise him as to whether he should stand for the election.

18.7 TEST FOR EQUALITY OF TWO MEANS (LARGE SAMPLES) (n_1+n_2-2 30)

For two independent random samples of large size n_1 and n_2, from two populations by the sample means x_1 and x_2 respectively to test equality two means we set up null hypothesis $H_1 : \mu_1 = \mu_2$ against alternative hypothesis $H_1 : \mu_1 \neq \mu_2$ (two tailed test)

Or $H_1 : \mu_1 > \mu_2$ (Right Tailed Test) or $H_1 : \mu_1 < \mu_2$ (Left tailed test)

For this test, we apply Z test as z = $\dfrac{\overline{x_1} - \overline{x_2}}{\sigma_{\overline{x_1} - \overline{x_2}}}$

Where $\sigma_{\overline{x_1}-\overline{x_2}} = \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

Where σ_1, σ_2 are two population sd's.

If instead of σ_1 and σ_2, sample of s_1 and s_2 are given then:

$$\sigma_{\overline{x_1}-\overline{x_2}} = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

Eg: (i) A simple sample of the height of 6400 Englishmen has a of 67.85 inches and SD of 2.56 inches, where a simple sample of heights of 1600 Austrians has a mean of 68.55 inches and a standard deviation of 2.52 . Do you think that the Austrians are on average taller than the Englishmen (use ignificance level)

Solution: $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 > \mu_2$ where μ_1 and μ_2 are denoting mean height of Austrian, and Englishmen respectively given : Austrian $n_1 = 1600, \overline{x_1} = 68.55, s_1 = 2.52$ and $n_1 = 6400, \overline{x_2} = 67.85, s_2 = 2.56$ Englishmen

$$\text{Where } \sigma_{\overline{x_1}-\overline{x_2}} = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = \sqrt{\frac{(2.52)^2}{1600} + \frac{(2.56)^2}{6400}}$$

We compute

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sigma_{\overline{x_1}-\overline{x_2}}} = \frac{68.55 - 67.85}{\sqrt{\frac{(2.52)^2}{1600} + \frac{(2.56)^2}{6400}}}$$

= 9.9

Taking =0.05, Z_α =2.58

As calculated $Z = 9.9 > Z_\alpha$

H_0 is rejected , Austrian's are on the average taller than the Englishmen.

Eg. (2) A man buys 50 electric bulbs of Philips and 50 electric bulbs of HMT. He finds that Philips bulbs gave an average life of 1500 hours with a standard deviation of 60 hours and 'HMT' bulbs gave an average life of 1512 hours with a standard deviation of 80 hours. If there is a significant difference in the mean life of the two makes of bulbs, (use =0.01)

Solution: We set $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 > \mu_2$ where μ_1 and μ_2 = mean life of Philips and HMT $n_2 = 50, \overline{x_2} = 1512, s_2 = 80$ electric bulbs respectively given. Philips $n_1 = 50, \overline{x_1} = 1500, s_1 = 60$ and HMT

We compute $$Z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{1500 - 1512}{\sqrt{\frac{(60)^2}{50} + \frac{(80)^2}{50}}}$$

$$= -0.848$$

Since Z $_{computed}$ =-0.848. $-0.848 > Z_{\frac{\alpha}{2}} (= -2.58)$ at $\alpha = 0.01$ level of significance H_0 is accepted. Mean life of the two marks is almost the same.

Test for equality of Two Means for small sample, Population (S.D.'s are unknown). ($n_1 + n_2 - 2<30$) Two populations are independent:

If the two population standard deviation are assumed to be equal with unbiased estimates of the common variance given by $S^2 = \frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2}$

Where s_1 and s_2 are the two sample standard deviations.

In this case we use Fisher's t distribution where $t = \frac{\overline{x_1} - \overline{x_2}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

Where d.o.f will be ($n_1 + n_2 - 2$)

18.7.1 TEST FOR EQUALITY OF TWO MEANS – PAIRED t TEST

This test used when for the same sample values which indicate an increase or decrease in the variable is given. Before and after values of the variable are normally given or the differences may be given directly.

Here d = ($X_{after} - X_{before}$)

We take $H_0 : \frac{\mu}{d} = 0$ against $H_1 : \frac{\mu}{d} \neq 0$

Or $H_1 : \frac{\mu}{d} > 0$ or $H_1 : \frac{\mu}{d} < 0$

$$\overline{d} = \sum d \Big/ n \quad \text{and} \quad s = \sqrt{\frac{\sum (d - \overline{d})^2}{n-1}} = \sqrt{\frac{1}{n-1}\left[\sum d^2 - \frac{(\sum d)^2}{n}\right]}$$

and $t = \dfrac{\bar{d}}{s/\sqrt{n}}$ (d. o. f n - 1)

Eg: (a) Two types of batteries are tested for their length of life and the following data are obtained:

	No of samples	Mean life in Hrs	Variance
type A	9	600	121
type B	8	640	144

Is there is a significant difference in the two means?

(Value of t for 15 degrees of freedom at 5% level is 2.131)

Solution: We set up null hypothesis $H_0 : (\mu_1 = \mu_2)$

Against $H_1 : (\mu_1 \neq \mu_2)$

Given $n_1 = 9, n_2 = 8, \bar{x}_1 = 600, \bar{x}_2 = 640$

$S_1^2 = 121, S_2^2 = 144$

The appropriate test statistic is Fisher's t which under H_0 follows t distribution with (n_1+n_2-2) d.o.f.

An estimate of the common but unknown sd (σ .....) is obtained from

$$s = \sqrt{\frac{9\times121+8\times144}{9+8-2}} = \sqrt{149.4} = 12.2$$

We compute t.. $\dfrac{600-640}{12.2\sqrt{\dfrac{1}{9}+\dfrac{1}{8}}} = \dfrac{-40}{12.2\times0.486}$

d.o.f. = $(n_1 + n_2 - 2)$ = 9 + 8 - 2 = 15

Computed $|t|$ = 6.7 which is larger than tabulated t (=2.13 for 15 d.o.f. of and α = 0.05)

x_{before}	110	120	123	132	125	Total
x_{after}	120	118	125	136	121	
d	-10	2	-2	-4	4	-10
d^2	100	4	4	16	16	140

So we reject c and accept H_1.

Our conclusion is there is significant difference in the two means.

(2) A IQ test was administered to 5 persons before and after they were trained. The results are given below:

	I	II	III	IV	V
IQ before training	110	120	123	132	125
IQ after training	120	118	125	136	121

Test whether there is any change in IQ after the training programme. (Given $t_{0.01,4}=4.6$)

Solution: This problem is a case for paired t test as the scores before training (x_{before}) and after training (x_{after}) are not independent, but the latter to be affected by the former i.e. they are correlated.

We take d = ($x_{before} - x_{after}$)

And set up null hypothesis $H_0 : \frac{\mu}{d} = 0$ against $H_1 : \frac{\mu}{d} \neq 0$

$$\therefore \bar{d} = \sum d \big/ n = -2$$

$$S = \sqrt{\frac{1}{n-1}\left[\sum d^2 - \frac{(\sum d)^2}{n}\right]}$$

$$= \sqrt{\frac{1}{4}\left[140 - \frac{100}{5}\right]} \quad = \sqrt{\frac{120}{4}} = \sqrt{30}$$

We computed $\wedge t = \dfrac{-2}{\frac{\sqrt{30}}{\sqrt{5}}} = \dfrac{-2}{\sqrt{6}} = \dfrac{-2\sqrt{6}}{6} = -0.82$

Critical region at 1% level of significance $|t| \geq 4.6$

Since the computed $|t| = 0.82$ is less than 4.6, $|t|_0$ cannot be rejected and we conclude that

there is no significant change in the mean IQ after the training programme.

18.8 TEST FOR EQUALITY OF PROPORTION

If P_1 and P_2 be the proportions in two large random samples of sizes n_1 and n_2 drawn respectively from two populations. To test whether the two proportions are equal or not we set hull hypothesis .

$H_0 : p_1 = p_2$ against $H_1 : p_1 \neq p_2$ or $H_1 : p_1 > p_2$ or $H_1 : p_1 < p_2$

Where P_1 and P_2 are two population proportions.

To examine, H_0 we use Z test as $Z = \dfrac{p_1 - p_2}{\sqrt{pq(\frac{1}{n_1} + \frac{1}{n_2})}}$

Where p = combined sample proportions = $\dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$ and q = 1 - p

Eg:1. A machine puts out 10 defective units in a sample of 200 units. After the overhauling the machine puts out 4 defective units in a sample of 100 units. Has the machine been improved. (use $\alpha = 0.05$)

Solution: Null hypothesis is that the proportions of defective before and after overhauling are equal. Alternative hypothesis is that the proportion of defectives has decreased after overhauling.

$H_0 : p_1 = p_2$ against $H_1 : p_1 > p_2$

we have $n_1 = 200, p_1 = \dfrac{10}{200} = 0.05$

$n_2 = 100, p_2 = \dfrac{4}{100} = 0.04$

Pooled proportion $\mathrm{P} = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \dfrac{10+4}{200+100} = 0.047$

Q = 1 − p =0.953

$$Z = \frac{p_1 - p_2}{\sqrt{pq(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{0.05 - 0.04}{\sqrt{0.047 \times 0.953\left(\frac{1}{200} + \frac{1}{100}\right)}} = 0.385$$

Since Z_{computed} (=0.385) is less than its critical value at then is accepted.

18.9 CHI SQUARE DISTRIBUTION (X^2 DISTRIBUTION)

Chi-Square Distribution has only one parameter called the degrees of freedom. (d.o.f). The shape of chi-square distribution depends on the number of degrees of freedom.

The important uses of x^2 test are:

(a) As a test for specified test

(b) As a test for goodness of fit

(c) As a test for independence of attributes

18.9.1 TEST FOR VARIANCE OF POPULATION

(i.e. for Specified sd.)

The test-statistics in this case is chi-square and is given by $x^2 = \frac{(n-1)s^2}{\sigma^2}$

(All notations have their usual meanings)

Hypothesis	Decision Rule
$H_0 : \sigma^2 = \sigma_0^2 against$ $H_1 : \sigma^2 \neq \sigma_0^2$	Accept H_0 if computed x² lies between $x^2_{1-\frac{\alpha}{2}}$ and $x^2_{\frac{\alpha}{2}}$ for (n- 1) d. o. f
$H_0 : \sigma^2 = \sigma_0^2 against$ $H_1 : \sigma^2 > \sigma_0^2$	Accept H_0 if computed x² is not more than x^2_{α} for (n – 1) d. o. f
$H_0 : \sigma^2 = \sigma_0^2 against$ $H_1 : \sigma^2 < \sigma_0^2$	Accept H_0 if computed $x^2_{1-\alpha}$ for (n – 1) d.o. f

Eg.1. Weights in kg of 10 students given below:

38,40,45,53,47,43,55,48,52,49

Can we say that variance of the distribution of weights of all students from which the above sample was drawn is equal to 20 square kgs?

(use 5% significance level, given that $x^2_{0.025} = 19.023$ and $x^2_{0.975} = 2.70$ for 9 d.o.f)

Solution: We set $H_0 : \sigma^2 = 20$

Against $H_1 : \sigma^2 \neq 20$

$x:$	$(x - \overline{x})$	$(x_i - \overline{x})^2$
38	-9	81
40	-7	49
45	-2	4
53	6	36

47	0	0
43	-4	16
55	8	64
48	1	1
52	5	25
49	2	4
$\sum x_i = 470$	0	780 = $\sum (x_i - \bar{x})^2$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{470}{10} = 47$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{280}{9}$$

Now, $x^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{9 \times \frac{280}{9}}{20} = 14$

As computed x^2 = 14which lies between $x^2_{0.975}(= 2.70)$ and $x^2_{0.025}(= 19.023)$. So we accept H_0 and infer that the population variance my be 20 sq kgs.

Eg.(2.) Specifications for the manufacture of parameter type of ornaments state that the variance in weight shall not exceed 0.015 mgm squared. A random sample of 15 such ornaments yields variance of 0.027. Can we say that specification are being met? $(use \alpha = 0.05)$

Solution: We set $H_0 : \sigma^2 = 0.015$ against

$H_1 : \sigma^2 > 0.015$

We compute $x^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(15-1) \times 0.027}{0.015} = 25.2$

For 14 d. o . f $x^2_\alpha = 23.68$ where $\alpha = 0.05$

Computed is more than the table value and we reject . Thus, the specifications are not being met.

(B) The Goodness of Fit Test

Pearsonian Chi-Square is $x^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$

Where E_i = Expected frequency

O_i = Observed frequency

K = Number of Categories

Eg: (1) A normal die is thrown 120 times and the following frequency distribution is observed.

Face :	1	2	3	4	5	6
Observed freq :	15	22	20	14	18	31

Can you regard the die as honest. (Givenx^2= 13.6 at 5% level of significance for 5 d.o.f. So we cannot reject the null hypothesis at 1% level of significance. The conclusion is that the data are in agreement with the hypothesis of an unbiased die.

Eg.(2) A Personal Manager is interested in trying to determine whether absenteeism is greater on one day of the week than on another. His records for the past year show the following sample distribution.

Day of Week :	Monday	Tuesday	Wednesday	Thursday	Friday
No. of absentees :	66	56	54	48	75

Test whether the absentee is uniformly distributed over the week.

Solution: We set up H_0 : Number of absence is uniformly distributed over the week against

H_1: Number of absence is not uniformly distributed over the week.

The number of absentees during a week are 300 and if absentism is equally probable on all days, then we should expect absentees on each day of the data as follows:

Category	O_i	E_i	O_i-E_i	$(O_i-E_i)^2$	$(O_i-E_i)^2/E_i$
Monday	66	60	6	36	0.60
Tuesday	57	60	-3	9	0.15
Wednesday	54	60	-6	36	0.60
Thursday	48	60	-12	144	2.40
Friday	75	60	-15	225	3.75
Total					7.50

The critical value of x^2 = 9.49 at $\alpha = 0.05$ and d.o.f. = (5-1) =4

As calculated value of x^2 = 7.5 is less than its critical value, the null hypothesis is accepted.

Test for Independence of Attributes

Frequencies related to two attributes may be tested to find whether an association between the attributes exist or not for the two –way table between the attributes the expected frequencies are calculated for each cell of the observed frequencies. The Chi-square value is less than evaluated using the formula

$$\hat{I}_i = \frac{\text{Row Total} \times \text{ColumnTotal}}{\text{Total frequency}}$$

and then $$x^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

The corresponding null hypothesis is H_0 (attributes are independent) against alternative hypothesis H_1 (attributes are not dependent)

Rejection Rule: If the calculated value of Chi-Square is greater than or equal to the tabulated value for (r-1)(c-1) d.o.f. where r = number of rows, and c=number of columns, then H_1rejected otherwise accepted.

Note: for 2×2 tables, the Chi-square formula for test of independence of attributes becomes simplified and can be used directly as $$x^2 = \frac{N(ad-bc)^2}{R_1R_2C_1C_2}$$

Where given table explains the notation,

2 X 2 table Row table

a	b	R_1
c	d	R_2

Column C_1C_2N = Total Frequency Total

Rejection Rule: The null hypothesis is rejected when the Chi-square value, is equal to or exceeds the tabulated value for (2-1)(2-1) =1 d.o.f.

Yates' Correction: For (2 x 2) table, there is always only one degree of freedom. To get better result of x^2 it is necessary to make a correction to the above formula and corrected formula is given by

$$x^2 = \frac{N\left\{|ad-bc| - \frac{N}{2}\right\}^2}{R_1R_2C_1C_2}$$

18.10 TESTS OF HYPOTHESIS ABOUT THE VARIANCE OF TWO POPULATIONS

While comparing the variance of two populations, data are collected from two independent random samples drawn each from the two different populations. If $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ are the unbi-ased estimate* on the basis of the sample variance s^2 is given by of the population variance based on sample sizes n_1 and n_2 respectively, the ratio $\hat{\sigma}_1^2 / \hat{\sigma}_2^2$ has a F-distribution with n_1 -1 d. o .f for the numerator and n_2-1 d.o.f. for the denominator. By using the suffix 1 with the larger sample variance. it is ensured, in case of two-tailed test $\hat{\sigma}_1^2 / \hat{\sigma}_2^2 \geq 1$. so that if a rejection of H_0 takes place, the rejection region appears in the upper-tail of tile F-distribution. A Summary of procedure is given below :

Hypotheses	Rejection region	Nature of H_1	Type of test	Decision rule	appears in
$H_O : \sigma_1^2 = \sigma_2^2$		H_1 carries $\neq$	Two-tailed	Accept H_0	
		Both tail			if $F \leq F_{a/2}$
$H_1 : \sigma_1^2 \neq \sigma_2^2$				Reject H_o if $F>F_{a/2}$	
$H_O : \sigma_1^2 \leq \sigma_2^2$		H_1 carries a	One tailed	Accept H_o	if $F \leq F_a$
		Right tail	strict inequality		
$H_1 : \sigma_1^2 > \sigma_2^2$		>		Reject H_o if $F>F_a$	

For one-tailed test H_o is set in such a way that the rejection region appears in the upper tail. This is done by a judicious numbering of the populations, so that the alternative hypothesis takes the form $H_1 : \hat{\sigma}_1^2 < \hat{\sigma}_2^2$. If H_1 is of the form, $\hat{\sigma}_1^2 < \hat{\sigma}_2^2$ we compute the ratio $\hat{\sigma}_2^2 / \hat{\sigma}_1^2$ (instead of $\hat{\sigma}_2^2 / \hat{\sigma}_1^2$) which also has a F distribution. but with n -1 d.o.f. for the numerator and n_1 - 1 d.o.f for the denominator. The critical value m the left tail of the F distribution is found from the table of critical values in the right tail of F distribution. For example, the reciprocal of $F_{0.095}$(d.o.f. d.o.f) in the right tail-will be $F_{0.095}$ (d.o.f . d.o.f } in the left-tail,

Example. The standard deviations calculated from two random samples of size 9and 13 are 2 and ! .9 respectively. May the samples be regarded as drawn from normal populations with the same standard deviation ? Value of $F_{0.025}$ from tables with d.o. f. 8 and 2 is 3.51.

Solution . We set $H_0 : \sigma_1^2 = \sigma_2^2$

$H1 : \sigma_1^2 \neq \sigma_2^2$

The unbiased estimate of the population variances in the two populations are given by

$\hat{\sigma}_1^2 = \frac{n_1}{n_1 - 1} s_1^2$ and $\hat{\sigma}_2^2 = \frac{n_1}{n_1 - 1} s_2^2$

Where n_1 . n_2 s_1 s_2 have their usual meaning. Here n_1=9, n_2 = 13, s_1 = 2, s_2 = 1.9

$$\therefore \hat{\sigma}_1^2 = \frac{9}{8} \times 4 = 4.5$$

$$\hat{\sigma}_2^2 = \frac{13}{12} \times 3.61 = 3.91$$

$$F = \frac{\hat{\sigma}_1^2}{\hat{\sigma}_2^2} = \frac{4.5}{3.91} = 1.15$$

Since 1.15 < 3.51. the given table value, the null hypothesis cannot be rejected

Example. Following results were obtained from two samples, each drawn from two different populations A and B.

Population	A	B
Sample	I	II
Sample size	n_1=25	n_2=17
Sample s.d.	s_1=3	s_2=2

Test the hypothesis that the variance of Brand A is more than that of Brand B.

(i) If $H_1 : \hat{\sigma}_1^2 < \hat{\sigma}_2^2$, can we reject the null hypothesis? Use $\alpha = 0.05$

Solution : (i) $H_O : \hat{\sigma}_1^2 \le \sigma\hat{}_2^2 \; and H_1 : \hat{\sigma}_1^2 > \sigma\hat{}_2^2$

$$\hat{\sigma}_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{25x9}{24} = 9.375$$

$$\hat{\sigma}_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{17 \times 4}{16} = 4.25$$

$$F = \frac{\hat{\sigma}_1^2}{\hat{\sigma}_2^2} = \frac{9.375}{4.25} = 2.206$$

.

From the table, $F_{0.05}$ = 2.24 for 24 and 16 d.o.f. Computed F (2.206) being less than $F_{0.05}$, we cannot reject H_O. i.e., variance of brand A is not more than that of brand B.

(ii) $F = \frac{4.25}{9.375} = 0.453$

F_{005} (24; 16) = 2.24 is in the right-tail of the distribution. By virtue of the reciprcal property. $F_{0.95}$ (I 6,24) = 1 /2.24 = 0.446 is the left-tail of the distribution. Computed F (0.453) being > F (0.446), we reject the null hypothesis.

Note: Had it been a case of two tailed test the table value for = 0.05 would have meant α =0.10.

18.11 ANALYSIS OF VARIANCE-A TEST FOR HOMOGENEITY OF MEAN,

This is one of the most elegant and versatile statistical techniques and finds wide application in determining whether or not the means of more than two populations are equal. Basically it is a procedure by which the variation in the data is split into different components attributable to several factors.

Underlying Assumptions

1. Each of the samples is a simple random sample.
2. Populations from which the samples arc selected are normally distributed.
3. Each oi ;hu populations has (he tame variance.
4. Each o-j /he samples is independent of other samples.

If, however the sample sizes are large enough, we do not need the assumption of normality. We shall explain the use of the F-distribution mainly by means of numerical exam-pies.

The technique of analysis of variance is referred to as ANOVA. A table showing the source of variation, the sum of squares, degrees of freedom, mean square (variance), and the formula for the F ratio is known as ANOVA table.

Let us adopt a standard arrangement of the data.

A sample observation x_0 has two subscripts. The subscript / denotes the row or sam-ple observation, while the subscript / denotes the column or population from which the observation came. Thus x_0 is the i-th? observation from the j-th population. That is x_0 is $N(\mu_1\sigma^2)$.

F-Test: This is another test of significance based on variance named after its pro-pounder R A Fisher. Its application is there in case of inference when two or more sam-ples are involved and the test is made on the basis of variances. The variances of random

Samples drawn from a normal universe have a distribution that is skewed positively; there-fore we have to have a separate distribution called F-distribution to test the significance based on variances.

Its application is normally in case of data in the form of a table with certain rows and columns. If there is only one attribute then it is called a one-way classification and it has as many columns as are the groups of samples of a given size. If, however, there are more than one characteristics then there is a table with a large number of rows and columns.

The various sums of squares involved in this are as follows: First consider the one way classified data

Typically the observations can be written in the following manner:

Treatments		Total
1: x_{11} x_{12} x_{13}......................................	x_1 n_1 (n_1 observation)	T_1
2: x_{21} x_{22} x_{23}......................................	x_2 n_2 (n_2 observation)	T_2
3: x_{31} x_{32} x_{33}	x_t n_t (n_t observation)	T_l
	Grand total	G

Let $N = n_1 + n_2 \ldots\ldots\ldots\ldots + n_t$

(i) Total sum/of the squares (TSS):

$$TSS = \sum_i \sum_j x^2{}_{ij} - \frac{G^2}{n}(i)$$

(Raw TSS-Correction Factor)

(ii) Sum of Squares due to treatments (SST):

$$SST = \sum \frac{T_j^2}{n_j} - \frac{G^2}{N}(ii)$$

(iii) Sum of Squares due to Error (SSE):
SSE = TSS - SST = (i) - (ii)

The null hypothesis is H_0 : $(\mu_1 = \mu_2 = ... = \mu_1)$

ANOVA. Table for One-Way classification

Source of Variable	Degrees of Freedom	Sum of Squares	Mean Sum of Squares	F
Between Treatments	t-1	SST	MST= $\frac{SST}{t-1}$	$\frac{MST}{MSE}$
Error	N-t	SSE	MSE= $\frac{SSE}{N-1}$	
Total	N-1	TSS		

If the derived value is more than the table value, the difference is significant and the null hypothesis is rejected. If the derived value is less than the table value the difference is not significant and the null hypothesis is accepted. The actual application will be clear from the following illustrations.

Example:

The three samples below have been obtained from normal populations with equal variances.

8	7	12
10	5	9
7	10	13
14	9	12
11	9	14

Test the hypothesis at 5% level that the population means are equal.

(The Table value of F at 5% level of significance for v_1 = 2 and v_2 = 12 is 3.88)

Solution:

We set H_0 : $\mu_1 = \mu_2 = \mu_3$

H_1 : At least two of the population means are unequal

Grand Total	8	7	12	27
Group mean $\overline{x_1}$	10	5	9	24
	7	10	13	30
	14	9	12	35
	11	9	14	34
	50	40	60	T=150
				Grand mean
	10	8	12	x = 10

The sum of squares for columns = SSC = $r\sum_{j}(\bar{x}_j - \bar{x})^2$ where $\bar{x}_j$ is the mean of the j^{th} sample, r is the number of rows, and x is the mean^j of the sample (columns) means.

$\therefore$ SSC=5 {$(10- 10)^2$ + $(8- 10)^2$+ $(12- 10)^2$-} = 5x8 = 40

The sum of squares for columns = MSC = $\frac{SSC}{c-1}$ where c stands for number of columns.

$\therefore$ MSC= $\frac{40}{20} = 20$

The sum of squares for the error = SSE = $\sum_{i}\sum_{j}(X_{ij} - X_j)^2$, where x_{ij} is the jth observation in the j^{th} sample and is the mean of the j th column.

$\therefore$ SSE = $(8 - 10)^2$+$(10\text{-}10)^2$+$(7\text{-}10)^2$+$(14\text{-}10)^2$+$(11\text{-}10)^2$

+ $(7\text{- }8)^2$ + $(5\text{ -}8)^2$ + $(I0\text{-}8)^2$ + $(9\text{-}8)^2$ + $(9\text{-}8)^2$

+ $(12\text{ - }I2)^2$+$(9\text{- }12)^2$ +$(13\text{- }!2)^2$+ $(12\text{- }12)^2$+ $(14\text{-}12)^2$

= 4 + 0 + 9 + 16+ I + 1+9 + 4+1 + 1+0 + 9+1+0 + 4 = 60

Variance within columns = MSE = $\frac{\text{SSE}}{\text{c(r-1)}} = \frac{60}{3\times4} = 5$

Total sum of squares of variations = SST = SSC + SSE = 40 + 60 =100

d.o.f. = c - 1 = 2: d.o.f.$_2$ = c (r - I) = 3 x 4 = 12

$$F=\frac{\text{MSC}}{\text{MSE}}=\frac{20}{5}=4$$

Source of Variation	Sum of Squares	Degrees of freedom	Mean square	F
Between samples (column means)	SSC = 40	d.o.f.$_1$ =2	MSC = 20	$F=\frac{\text{MSC}}{\text{MSE}}$
Within samples	SSE=60	d.o.f.$_2$ =12	MSE = 5	= 4
Total	SST = 100	cr – 1 = 14		

At 5% level the table value of F for v_1 = 2 and $v_{2=}$ 12 is given to be 3.88. but the computed value of F is greater than this table value. We, therefore, reject H_O. and conclude that the population means are not equal.

Alternative approach for computation of SSC. SST; SSE.

Correction factor. C = $\frac{T^2}{rc}$. where T is the grand total of values in all the samples, r is the number of rows, and t. is the number of columns.

$$= \frac{150x150}{5x3} = 1500$$

Now SSC = $\frac{\sum_j T_J^2}{r}$ -C , where T stands for the total of the j[1h] column.

$$\therefore \text{SST} = \sum_I \sum_J X^2 Y\text{-}C = (8^2+10^2+7^2+14^2+11^2+7^2+5^2+10^2+9^2+9^2+12^2+9^2+13^2+12^2+14^2)\text{-}1500$$

= 1600 − 1500 = 100

SSE = SST-SSC= 100-40 = 60

18.12 ANALYSIS OF VARIANCE IN MANIFOLD CLASSIFICATION:

In manifold classification there are two or more characteristics which are considered.

The table dealing with data on such grouping has a number of columns and rows. The analysis in that case will get extended to include the sum of squares between rows, which was not there in one-way classification.

In a two-way classification, the observations are as follows.

Rows	Columns					
0	1	2	....	c		
1	x_{11}	x_{12}	....	x_{1c}	R_1	
2	x_{21}	x_{22}	....	x_{2c}	R_2	
..	..	..	....	....		
r	x_{r1}	x_{r2}	....	x_{rc}	R_r	
Total	c_1	c_2	....	c_c	G	(Grand Total

Let N = rc and the two null hypotheses are

H_{01} : (Row means are equal)

H_{02} : (column means are equal)

(i) Total sum of squares (TSS):

$$\text{TTS} = \sum_i \sum_j x_{ij}^2 - \frac{G^2}{N}$$

(ii) Sum of Squares due to Columns (SSC):

$$SSC=\sum_{j=i}^{C}\frac{C^2{}_jG^2}{rN}$$

(iii) Sum of Squares due to Rows (SSR):

$$SSR=\sum_{j=i}^{C}\frac{R^2{}_c}{c}-\frac{G_2}{N}$$

(iv) Sum of Squares due to Error (SSE)

SSE = TSS-SSC-SSR

ANOVA Table for two-way Classification

Source of variation	Sum of Squares	d.o.f.	Mean squares	Variance ratio
Between columns	SSC	(c-1)	$\frac{SSC}{c-1}$ = MSC	$F_C \frac{MSC}{MSE}$
Between Rows	SSR	(r-1)	$\frac{SSC}{r-1}$=MSR	$F_R \frac{MSC}{MSE}$ c
Error	SSR	(c-1)(r-1)	$\frac{SSC}{(r-1)(c-1)}$=MSR	
Total	**SST**	**cr - 1**		

Example: A farmer applies three types of fertilizers on 4 separate plots. The figure on yield per acre are tabulated below:

	Yield				Total
Fertilisers / Plots	A	B	C	D	T
Nitrogen	6	4	8	6	24
Potash	7	6	6	9	28
Phosphates	8	5	10	9	22
total	21	15	24	24	84

Find out if the plots are materially different in fertility as also if the three fertilizers make any material difference in yields.

Solution: Let us first determine the correction factor,

$$C=\frac{T^2}{rc}=\frac{(84)^2}{12}=558$$

The sum of squares between columns is SSC = $\sum \frac{(\sum x_j)^2}{n_j}$-C

$$=\left[\frac{(21)^{2=}}{3}+\frac{(15)^2}{3}+\frac{(24)^2}{3}+\frac{(24)^2}{3}\right]-588=\frac{1818}{3}-588=606-588=18$$

The sum of squares between rows is, SSR = $\sum_{i=1}^{r} T_I^2/c\text{-}C$

$$=[\frac{(24)^2}{4}+\frac{(28)^2}{4}+\frac{(32)^2}{4}]-588=[\frac{576}{4}+\frac{784}{4}+\frac{1024}{4}]-588,=2384/4-588=596-588=8$$

The total sum of squares is. SST = $\sum_{i-i}^{R}\sum_{j=1}^{c} x^2_{ij}$-C

$$=[(6)^2+(7)^2+(8)^2+(4)^2+(6)^2+(5)^2+(6)^2+(10)^2+(6)^2+(9)^2+(9)^2]-588$$
$$=[36+49+64+16+36+25+64+36+100+36+36+81+81]-588$$
$$=624-588=36$$

The error sum of squares is. SSE = SST - (SSR + SSC) = 36 - (8 + 18) = 10.

The table for Analysis of Variance.

Source of variation	Sum of squares	Degrees of freedom	Mean squares	F-Ratios
Between plots	18	4.1=3	MSC = 18/(4-1)=6	Fc= $\frac{6}{1.667}=3.6$
Between fertilizers	8	3-1=2	MSR = 8/(3-1) = 4	$F_R = \frac{4}{1.667}=2.4$
Error (SSE)	10	3 X 2 =6	MSE=10/6=1.667	
Total	36		cr – 1 = 12 – 1 = 11	

The F-value for (3,6) degrees of freedom is 7.76 and for (2,6) degrees of freedom is 5.14both at 5% level of significance The computed F values being lower, they do not show any significant difference, and whatever difference exists is due to sampling error.

SELF-EXAMINATION QUESTIONS

1. A supplier of components to the electronic industry makes a sophisticated product which sometimes fails immediately it is used He controls his manufacturing process so that the proportion of faulty products is supposed to be only 5%. Out of 400 supplies in one batch. 26 prove to be faulty. Has the process gone out of control to produce too man) faulty components?

2 Ten objects are chosen at random from a large population and their weights are found to be in gms. 63, 63. 64, 65. 66.69. 70. 70 and 71. In the light of the above data discuss the suggestion that the mean weight in the universe is 65 gms.

3. On a certain day, 74 trains were arriving on time at Delhi station during the rush hour and 83 were late. At New Delhi station there were 65 on time and 107 late. Is there any difference in the proportions arriving on time at the two stations?

4. Set up ANOVA table for the following per hectre yield for three varieties of wheat, each grown on four plots and carry out the test for the varieties.

Per hectare yield (in hundred Kgs.)

Plot of land	Variety of Wheat		
	A1	A2	A3
1	6	5	5
2	7	5	4
3	3	3	3
4	8	7	4

Answers

1. $Z_{computed}$ = 1.37, manufacturing process in probably not out of control ;
2. $t_{computed}$ = 2.02. Mean weight of the universe is likely to be 65 gms;
3. Proportion of trains arriving on time at the two stations are same.
4.

Source of Variation	Sum of squares	d.o.f.	Mean square	Variance ratio
Between varieties (Column means)	8(SSC)	c-1=2	4(MSC)	$F = \frac{MSC}{MSE}$
Within samples (Error)	24(SSE)	c(r-1) = 9	2.67(MSE)	=4/2.67=1.5

Chi Square Test.

1. The theory predicts the proportion of beans in the 4 groups A, B, C and D should be 9:3:3:1. In an experiment among 1600 beans, the numbers in the 4 groups were 882, 313, 287 and 118. Does the experimental result support the theory?

Answer. $\chi^2 = 4.72 < 7.8$, Yes

2. The following table gives the number of aircraft accidents that occur during various days of the week. Find whether the accidents are uniformly distributed over the weekdays.

Days	:	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
No of accident	:	14	16	8	12	11	9	14

Answer. $\chi^2 = 4.16 < 12.59$, Yes]

3. Three Hundred digits were chosen at random from a set of tables. The frequencies of the digits were as follows:

Digits	0	1	2	3	4	5	6	7	8	9
Frequency	28	29	33	31	26	35	32	30	31	25

Using χ^2 test assess the hypothesis that the digits were distributed in equal number in the table.

Answer: Yes, $\chi^2 = 2.864 < 16.92$]

4. A die tossed 120 times with the following results.

Face	1	2	3	4	5	6	Total
Frequency	30	25	18	10	22	15	120

Test the hypothesis that the dice is unbiased. [Ans. No, $\chi^2 = 12.90 > 11.07$]

5. A Bird watcher sitting in a park has spotted a number of birds belonging to 5 categories. The exact classification is given below:

Category	:	1	2	3	4	5	6
Frequency	:	6	7	13	17	6	5

Examine whether or not the data is compatible with the assumption that This particular is visited by birds belonging to these six categories in the Proportion 1: 1: 2: 3: 1: 1.

Yes, $\chi^2 = 0.47 < 11.07$]

6. From the table given below, test whether the colour of the son's eyes is associated with that of father's eyes.

		Eye colour of sons	
		Not light	Light
Eye colour of fathers	Not light	230	148
	Light	151	471

(Given $\chi^2 = 3.84$ at 1df &5% significance)

Answer: 133.39 > 3.84, there is relation]

7. The following are collected on two characters.

	Cinema goers	Non cinema goers
Literate	92	48
Illiterate	208	52

Based on this can you conclude that there is a relation between habit of cinema going and literacy (Given 5% points of χ^2 distribution are 3.841, 5.991 and 9.488 for degrees of freedom 1, 2 and 4 respectively)

Answer: $\chi^2 = 9.9 > 3.84$, Yes there is relation]

8. Calculate the expected frequencies for the following data presuming the two attributes viz. conditions of home and condition of child are independent.

Condition	Condition of home	
of child	Clean	Dirty
Clean	70	50
Fairly clean	80	20
Dirty	35	45

Answer: $\chi^2 = 25.633 > 5.99$, there is an association between the two]

9. In an examination on immunization of cattle from tuberculosis the following result were obtained.

	Affected	Unaffected
Inoculated	12	28
Not inoculated	13	7

Examine the effect of vaccine in controlling the incidence of the disease.

Answer: χ^2 6.729 > 3.84 , Yes.]

10. Two sample polls of votes for two candidates A and B for public office are taken, one from among residents of rural areas and the other from urban areas. The results are given in the table. Examine whether the nature of the area is related to voting preference in this election.

	Vote for		
Area	A	B	Total
Rural	620	380	1000
Urban	550	450	1000
Total	1170	830	2000

Answer: $\chi^2 = 10.089 > 3.84$, Nature of area is related to voting]

11. Two groups of 100 people each were taken for testing the use of a vaccine.

15 persons contracted the disease out of the inoculated persons while 25 contracted the disease in the other group. Test the efficiency of vaccine using χ^2 – test.

Answer. Not effective, χ^2 =3.125 < 3.84]

12. In a survey of 200 boys, of which 75 were intelligent. 40 had skilled fathers, while 85 of the unintelligent boys had unskilled fathers. Do these data support the hypothesis that skilled fathers have intelligent boys.

Answer Yes, χ^2 = 8.89 > 3.84]

13. Out of 800 persons 25% were literates and 300 had enjoyed T.V. programmes. 30% of those who had not enjoyed T.V. Programmes wereliterate. Test at 5% level of significance whether T.V. Programmes influence literacy.

Degree of freedom	:	1	2	3	4
Chi – Square at 5% level	:	3.841	5.999	7.815	9.448

Answer: χ^2 = 17.78 (H_o is rejected T.V. program significantly influence literacy]

14. Out of 8000 graduates in a town 800 are females. Out of 1600 graduates employees 120 are females. Test whether there is any distinction made in the appointment on the basis of female or male criterion.

Answer: [χ^2 = 13.89. Yes distinction is made]

Test of Hypothesis Exercise:

1. Two types of new cars produced in India are tested for petrol mileage. One group consisting of 36 cars averaged 14 kms per litre. While the other group consisting of 72 cars averaged 12.5 kms per litre. Test whether there is any existence of significant difference in the petrol consumption of these two types of cars. (use)

Answer: There is significant difference in petrol consumption of the two types of new cars.

2. 500 units from a factory are inspected and 12 are found to be defective. 800 units from another factory are inspected and 12 are found to be defective Can it be concluded that at 5% level of significance production at the second factory is better than in first factory.

Answer: Production in second factory is better than in the first factory.

3. Two independent random sample were taken two normal population and the following information were obtained:

	Population 1	Population II
Sample size	10	12
Sample mean	20	27
Population sd	8	6

It is likely that the mean of the first population is less than that of the second. (use 1% of level of significance.) (Answer : No, Z = - 2.28)

TEST OF HYPOTHESIS OR TEST OF SIGNIFICANCE

EXERCISE

1. A manufacturer of golf balls introduces a new type of ball (brand 2) which he claims is superior to the previous brand (brand 1) in terms of the distance it will carry under identical playing conditions. To test this claim a golfer, selected at random, hits 35 shots using the old brand (brand 1) while another randomly selected golfer hits 40 shots with new brand (brand 2).The distance in meters for each brand are recorded and summarized as follows (the raw data are shown in the computer solution below);

Brand 1	Brand 2
$n_1 = 35$	$n_2 = 40$
=202.9 meters	=223.7 meters
$s_1 = 29.9$ meters	$s_2 = 35.0$ meters

Test the hypothesis;

(a). that the balls are no different , (b). that brand 1 is inferior to brand 2.

2. The means of two large samples of sizes 1000 and 2000 are 67.5 and 68.0 respectively, Test the equality of means of the two populations each with s.d. 2.5.

3. Two independent random samples were taken from two normal populations and the following information is given:

	Population I	Population II
Sample size	$n_1 = 10$	$n_2 = 12$
Sample mean		
Population s.d.		

Is it likely that the mean of Population I is smaller than that of Population II?

(Use 1% level of significance). Also find 99% confidence limits for the difference of population means.

4. Two types of batteries are tested for their length of life and the following data are obtained:

No. of sample	Mean life in Hours	Variance	
Type A	9	600	121
Type B	8	640	144

Is there a significant difference in the two means? Value of t for 15 degrees of freedom at 5% level is 2.131.

5. A machine produces 16 defective in a batch of 500. After overheating it produced 3 defective in a batch of 100. Has the machine improved? (Use 5% level of significance).

6. In a large city A, 20 per cent of a random sample of 900 school children had defective eye – sight. In another large city B, 15 percent of a random sample of 1600 children had the same defect. In this difference between the two proportions significant?

7. In a certain district A , 450 persons were considered regular consumers of Tea out of a sample of 1000 persons. In another district B. 400 were regular consumers of Tea out of a sample of 800 persons. Do these facts reveal a significant difference between the two districts as far as tea – drinking habit is concerned ? (Use 5% level)

8. A set of 10 students is selected at random from a college and they were given an intensive coaching on mathematics and their scores are given below:

Scores before coaching	10	8	7	9	8	10	9	6	7	8
Score after training	12	8	8	10	8	11	9	8	9	9

Do you think that there is any significant effect due to coaching?

Answer: Yes, t =3.87 >2.262]

9. Memory Capacity of 9 students was tested before and after training. State whether the training was effective or not from the following score:

Students	1	2	3	4	5	6	7	8	9
Before	10	15	9	3	7	12	16	17	4
After	12	17	8	5	6	11	18	20	3

Answer: No,t =1.3<1.86]

10. A drug to 10 patients and the increments in their blood pressure were recorded as 3, 6, - 2, 4, - 3, 4, 6, 0, 0, 2. Is it reasonable to believe that the drug has no effect on change of blood pressure ? [Use 5% value of t for 9df = 2.26.].

Answer: Yes, t = 2 < 2.26]

11. Marks obtained by 10students are given below:

58	60	65	73	67	63	75	68	72	69

Can we say that sd of the distribution of marks of all students from which the above sample was drawn is equal to 4.47? [Use 5% significance level, given that $\chi^2_{0.0\,25} = 19.023$ and $\chi^2_{0.975} = 2.7$ for 9 df]

Answer: Yes, $\chi^2 = 14.01 > \chi_{0.975}{}^2$]

12. A manufacturer claims that variance in any of his lot of items cannot be more than 1 cm^2. A sample of 25 items has a variance 1.2 cm^2. Test the claim at 5% level. [Given $\chi^2_{0.05,\ 24} = 36.415$]

Answer: $\chi^2 = 28.8<36.415$, H_1; > t is rejected i.e. , claim is justified]

13. A random sample of size 20 from a normal population gives a sample mean of 42 and sample sd of 6. Test the hypothesis that population sd is 9.

Answer: Yes, H_o is accepted]

14. Specification for the manufacturing of a particular type of product state that variance in weight not exceed 1 gm. A random sample of 9 such product yields a variance of 2.5 gm. Can we say that specifications are being met?

    [Use = 0.05]

Answer: No, H_o is rejected.]

15 The standard deviation calculated from two random samples of size 9 and 13 are 2 and 1.9 respectively. May the sample be drawn from the normal population with same sd.? [Given $F_{0.05}$ (8, 12) = 2.85, $F_{0.05}$ (12, 8) = 3.28]

Answer: F = 115, H_0 ; ( ) is accepted.]

16. In a sample of 8 observations the sum of deviations of items from the mean was 94.5. In another sample of 10 observations, the value was found to be 101.7. Test whether the difference is significant at 10% level. [Given $f_{0.05}$ (7, 9) = 3.29 and $f_{0.05}$ (8, 10) = 3.07]

Answer: H_0 is accepted, Not significant]

17. Two sources of raw materials are under consideration by a company. Both sources seem to have similar characteristics, but the company is not sure about their respective uniformity. A sample of ten lots from source A yields a variance of 225 and sample of eleven lots from source B yields a variance of 200. Is it likely that the variance of source A is significantly greater than the variance of source B? [Use = 0.01]

Answer: Yes, F = 1.136]

18. Specifications for the manufacture of a particular type of ornament state that the variance in weight shall not exceed 0.015 mg. squared. A random sample of 15 such ornaments yields a variance of 0.027. Can we say that specifications are not being met? [The value of χ^2 is 23.685 for 14d. of f. at 0.05 level]

Answer: No. H_1 is accepted χ^2 = 27]

19. Weight in Kg. of 10 students are 38, 40, 45, 53, 47, 43, 55, 48, 52 and 49. Can we say that the variance of the distribution of weight of all students from which the above sample of 10 students was drawn, is equal to 20squares Kgs?

Answer: H_1 is accepted, χ^2 =14]

20. A random sample of size nine is drawn from a normal population constitutes the observation 163, 164, 165, 166, 167, 168, 168, 170 and 172. Test the hypothesis that the population S.D. is 12, its mean is being given to be 166.

Answer: H_1 is accepted , χ^2 = 0.5625]